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An Electromechanical Coning Damper, Rebound Project, Model DSV-5

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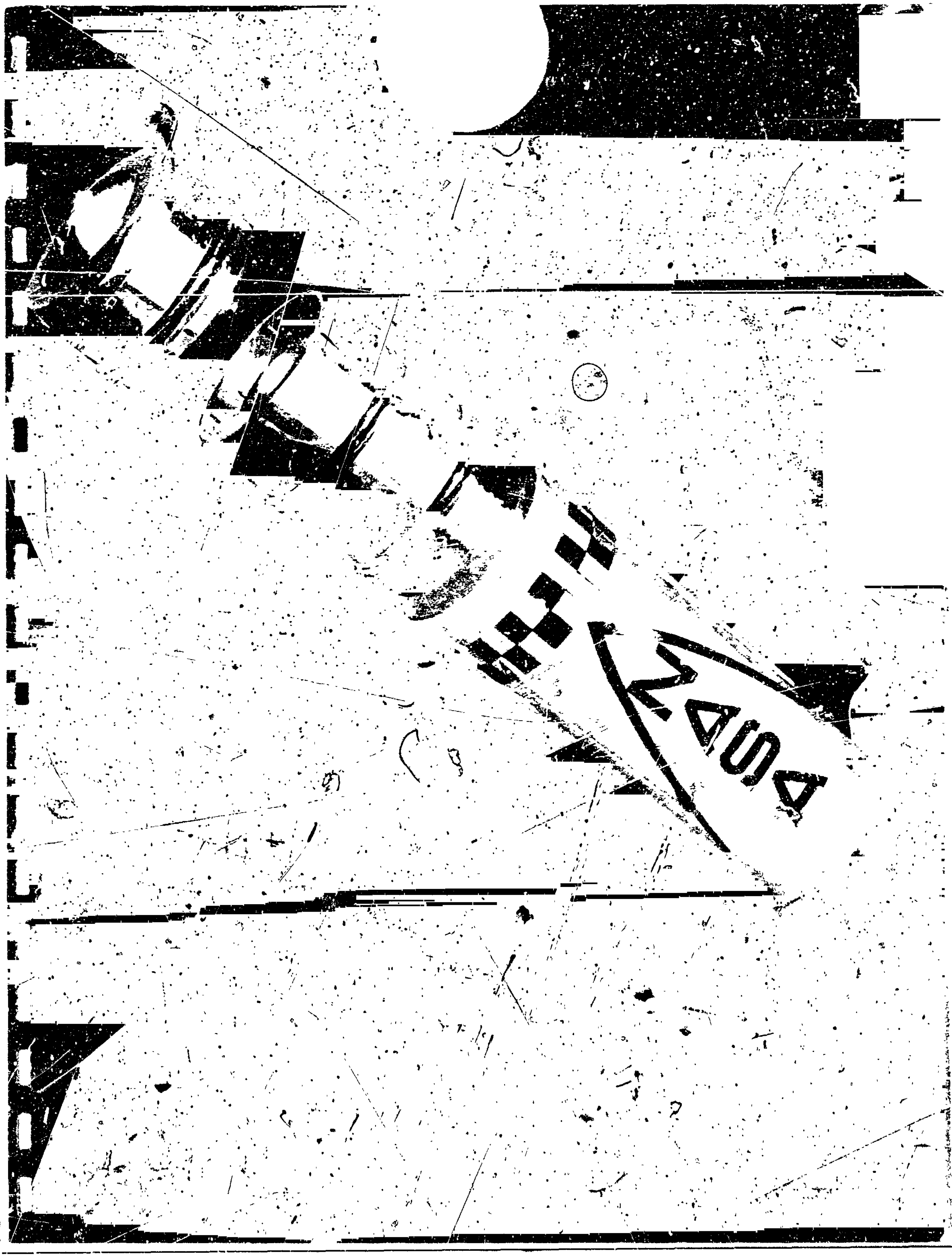
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ABSTRACT

This report derives a control equation for an electromechanical coning damper. The damper consists of a wheel mounted in a spinning satellite and driven by a signal derived from an accelerometer mounted on the spinning body. Since the damping is done internally, the angular momentum is unchanged and a constant direction in space may be maintained.

The damper may be used for any roll-to-pitch inertia ratio except unity.

Author

PREFACE

This report is prepared by Douglas Aircraft Company, Inc. (DAC). It details the electromechanical coning damper studies as required under National Aeronautics and Space Administration (NASA) contract NAS5-1294. The studies cover a period from September 1961 through January 1962.

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1.0 INTRODUCTION

A free spinning satellite will undergo predictable motion based on the Euler equations. A first integral of the Euler equations (energy) coupled with the constancy of the angular momentum vector may be used to describe the periodic motion of the satellite known as coning.

If no external moments are applied to the spinning satellite but the energy changes, the coning motion will change. A control over the coning motion is possible by means of control over the energy of the system.

A cylindrical satellite spinning about its long axis will upon losing energy increase its coning angle. The loss of energy comes as a result of coning forced motion of flexible parts inside the satellite as well as the flexing of the satellite itself.

An alternate mode of the Rebound injection scheme calls for spin stabilization of the spacecraft free of the Agena for extended periods of time. The shape of the spacecraft and the unavoidable energy loss dictates the need for a method of reducing the cone angle.

An energy control system to reduce the coning motion of a long slender spinning satellite is the subject of this report.

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2.0 SUMMARY

Development of the equations has shown that a spinning rigid body which is coning may be controlled by a wheel internal to it, rotating in a prescribed manner relative to the inertial motion of the body.

The proper relationship between the rotation of the wheel and the body may be sensed by an accelerometer mounted in the plane of the cg . The accelerometer should be mounted parallel to the centerline at a location 90° away from the centerline of the wheel.

The effect of the coning damper is to constrain the rate of change of the coning angle to be always negative.

3.0 RELATIONSHIP BETWEEN CONING ANGLE AND ENERGY

Let us derive the equations of motion for a spinning satellite and observe the relationship between the coning angle and the system energy.

Referring to Figure 1 and assuming there is no wheel in the satellite we may say:

$$\vec{P} = P_1 \hat{1} + P_2 \hat{2} + P_3 \hat{3}$$

$$\vec{\omega} = \omega_1 \hat{1} + \omega_2 \hat{2} + \omega_3 \hat{3}$$

$$P_1 = \omega_1 I_R \,;\; P_2 = \omega_2 I \,;\; P_3 = \omega_3 I$$

Now

$$\vec{M} = \dot{\vec{P}} = \vec{\dot{P}} + \vec{\omega} \times \vec{P}$$

So that

$$\begin{aligned} M_1 &= I_R \dot{\omega}_1 \\ M_2 &= I \dot{\omega}_2 - (I \omega_1 \omega_3 - I_R \omega_1 \omega_3) \\ M_3 &= I \dot{\omega}_3 + (I \omega_1 \omega_2 - I_R \omega_1 \omega_2) \end{aligned} \qquad (1)$$

Equations (1) are the so called Euler equations for a symmetrical rigid body.

If we define

$$E = \frac{1}{2} \left\{ \omega_1^2 I_R + \omega_2^2 I + \omega_3^2 I \right\}$$

Then

$$2E = I_R \omega_1^2 + I (\omega_2^2 + \omega_3^2)$$

$$P^2 = I_R^2 \omega_1^2 + I^2 (\omega_2^2 + \omega_3^2)$$

Eliminating $\omega_2^2 + \omega_3^2$ between the two equations

$$\frac{2E}{I} - R \omega_1^2 = \frac{P^2}{I^2} - R^2 \omega_1^2$$

$$2E = \frac{P^2}{I} + I_R (1 - R) \omega_1^2$$

$$I_R^2 \omega_1^2 = \frac{R (2EI - P^2)}{1 - R}$$

and $\cos \theta = \dfrac{I_R \omega_1}{|P|}$ $\qquad\qquad$ θ = cone angle

$$\cos\theta = \frac{1}{|P|}\sqrt{\frac{R(2EI - P^2)}{(1-R)}}$$

differentiating remembering P is a constant.

$$-\sin\theta\, d\theta = \frac{1}{2|P|}\sqrt{\frac{(1-R)}{R(2EI-P^2)}}\ \frac{2RI}{(1-R)}\ dE$$

From this expression it may be seen that for $R < 1$ (a slender satellite) an increase in E will result in a decrease in θ the cone angle.

4.0 EQUATIONS OF AN ACTIVE CONING DAMPER

The coning damper proposed consists of a rotating wheel mounted so that its spin axis is normal to the centerline of the vehicle and, for simplicity of this derivation, placed so that its c.g. is at the vehicle c.g.

Let us then derive the Euler equations for the spinning satellite with a wheel attached to it. The moment of inertia of the wheel (assumed rigidly attached) is absorbed in the vehicle moments of inertia (I, and I_R).

Referring to Figure 1.

$$P_1 = I_R \omega_1$$

$$P_2 = I\omega_2 + I_1 \Omega$$

$$P_3 = I\omega_3$$

where I_1 is the roll inertia of the wheel

again

$$\vec{M} = \dot{\vec{P}} = \dot{\vec{P}} + \vec{\omega} \times \vec{P}$$

so

$$M_1 = \dot{P}_1 + (\vec{\omega} \times \vec{P}) \cdot \hat{1}$$

$$M_2 = \dot{P}_2 + (\vec{\omega} \times \vec{P}) \cdot \hat{2}$$

$$M_3 = \dot{P}_3 + (\vec{\omega} \times \vec{P}) \cdot \hat{3}$$

which then becomes

$$M_1 = I_R \dot{\omega}_1 + \omega_2 P_3 - \omega_3 P_2$$

$$M_2 = I\dot{\omega}_2 - \omega_1 P_3 + \omega_3 P_1 + I_1 \dot{\Omega}$$

$$M_3 = I\dot{\omega}_3 + (\omega_1 P_2 - \omega_2 P_1)$$

or

$$M_1 = I_R \dot{\omega}_1 - I\omega_3 \Omega$$

$$M_2 = I\dot{\omega}_2 - (I - I_R)\omega_1 \omega_3 + I_1 \dot{\Omega}$$

$$M_3 = I\dot{\omega}_3 + (I - I_R)\omega_1 \omega_2 + I_1 \omega_1 \Omega$$

These equations then describe the motion for a spinning satellite with a rotating wheel.

5.0 CONING ANGLE REDUCTION BY THE CONING DAMPER

Proceeding as before

$$2E = I_R\omega_1^2 + I\omega_3^2 + (I - I_1)\,\omega_2^2 + I_1\,(\omega_2 + \Omega)^2$$

$$\dot{E} = I_R\dot{\omega}_1\omega_1 + I\,\dot{\omega}_2\omega_2 + I\,\dot{\omega}_3\omega_3 + I_1\dot{\Omega}\,\Omega + I_1\,(\dot{\omega}_2\Omega + \omega_2\dot{\Omega})$$

or after combining with Euler equations (Section 4)

$$\dot{E} = I_1\,\Omega\,(\dot{\Omega} + \dot{\omega}_2)$$

$$\frac{2E}{I} = \frac{I_R}{I}\,\omega_1^2 + \omega_2^2 + \omega_3^2 + \frac{I_1}{I}\,\Omega^2 + \frac{2I_1}{I}\,\omega_2\Omega$$

$$\frac{P^2}{I^2} = \frac{I_R^2}{I^2}\,\omega_1^2 + \omega_2^2 + \omega_3^2 + \frac{I_1^2}{I}\,\Omega^2 + 2\,\frac{I_1}{I}\,\Omega$$

then

$$2EI - P^2 = I^2\left\{\frac{2E}{I} - \frac{P^2}{I^2}\right\} = \omega_1^2\,I_R^2\left(\frac{I}{I_R} - 1\right) + \Omega^2\,(II_1 - I_1^2)$$

$$\omega_1 I_R = \left\{\frac{(2EI - P^2) - \Omega^2\,(I\,I - I_1^2)}{\left(\frac{1}{R} - 1\right)}\right\}^{\frac{1}{2}}$$

If P is constant,

$$\cos\theta = \frac{\omega_1 I_R}{|P|} \quad -\sin\theta\,\frac{d\theta}{dt} = \frac{1}{|P|}\,\frac{d}{dt}\,(I_R\,\omega_1)$$

$$\frac{d}{dt}\,(I_R\,\omega_1) = \frac{1}{2\,\omega_1 I_R}\left\{\frac{2I\dot{E} - 2\,\Omega\,\dot{\Omega}\,(I_1 I - I_1^2)}{\left(\frac{1-R}{R}\right)}\right\}$$

Substituting in $\dot{E}$;

$$-\sin\theta\,\frac{d\theta}{dt} = \frac{1}{\omega_1\,I_R|P|}\left\{\frac{II_1\,\Omega\,(\dot{\Omega} + \dot{\omega}_2) - \Omega\dot{\Omega}\,(I_1 I - I_1^2)}{\left(\frac{1-R}{R}\right)}\right\}$$

$$-\sin\theta\,\frac{d\theta}{dt} = \frac{1}{\omega_1 I_R|P|}\left\{\frac{I_1\Omega\,(I\,\dot{\omega}_2 + \dot{\Omega}\,I_1)}{\left(\frac{1-R}{R}\right)}\right\}$$

which upon substituting from the second Euler equation

$$-\sin\theta\,\frac{d\theta}{dt} = \frac{1}{\omega_1 I_R|P|}\quad\frac{I_1\Omega\,(I - I_R)\,\omega_1\,\omega_3}{\left(\frac{1-R}{R}\right)}$$

so

$$-\sin\theta\,\frac{d\theta}{dt} = \frac{1}{|P|}\,\left(\frac{I - I_R}{I_R}\right)\,I_1\Omega\,\omega_3\,\frac{R}{(1-R)}$$

and

$$-\sin\theta \frac{d\theta}{dt} = \frac{1}{|P|} I_1 \Omega \omega_3$$

therefore

$$-\sin\theta \frac{d\theta}{dt} = \frac{I_1 \Omega \omega_3}{|P|}$$

we may then use either of the two equations

$$-\frac{d\theta}{dt} = \frac{1}{\omega_1 I_R |P| \sin\theta} \left(\frac{R}{1-R}\right) \left[I_1 \Omega (I \dot{\omega}_2 + \dot{\Omega} I_1)\right] \tag{5-1}$$

or

$$-\frac{d\theta}{dt} = \frac{I_1 \Omega \omega_3}{|P| \sin\theta}$$

To mechanize a system where $\dot{\theta}$ is always negative (a coning angle reduction).

The first of these equations was chosen for mechanization because of the ease of obtaining $\dot{\omega}_2$ and the amplifying factor $R/_{1-R}$.

6.0 ACCELEROMETER AS A CONING SENSOR

If we place an accelerometer at the edge of a flat disk in Figure 2, we may directly develop the acceleration sensed by the accelerometer

$$\vec{a} = \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

Letting r be a vector $r_2 = 0, \quad r_3 = r, \quad r_1 = 0$

$$a_1 = -r(\dot{\omega}_2 - \omega_1 \omega_3)$$

An expression for $\omega_1 \omega_3$ from the modified Euler equations of Section 4 is given as:

$$\omega_1 \omega_3 = \frac{I}{I - I_R} \dot{\omega}_2 + \frac{I_1}{(I - I_R)} \dot{\Omega} .$$

then

$$a_1 = -r\left(\dot{\omega}_2 - \left(\frac{I}{I - I_R}\right)\dot{\omega}_2 - \frac{I_1}{I - I_R} \dot{\Omega}\right) .$$

or

$$a_1 = -r\left(\dot{\omega}_2\left[1 - \frac{I}{I - I_R}\right] - \dot{\Omega} \frac{I_1}{I - I_R}\right)$$

or

$$a_1 = -r\left[\dot{\omega}_2\left(\frac{-I_R}{I - I_R}\right) - \dot{\Omega} \frac{I_1}{I - I_R}\right]$$

and if $R = \frac{I_R}{I}$

$$a_1 = \frac{-rR}{(I - I_R)}\left[-I\dot{\omega}_2 - \frac{I_1}{R}\dot{\Omega}\right]$$

or

$$a_1 = \frac{rR}{(I \cdot I_R)}\left[I\dot{\omega}_2 + \frac{I_1}{R}\dot{\Omega}\right]$$

$$a_1 = \frac{r}{I}\left(\frac{R}{1 - R}\right)\left[I\dot{\omega}_2 + \frac{I_1}{R}\dot{\Omega}\right]$$

or from Equation 5-1 if R is nearly 1

$$\frac{-d\theta}{dt} \cong \frac{1}{\omega_1 I_R} \frac{I I_1}{|P| \sin\theta\, r} a_1 \Omega$$

so a control equation of the form $\Omega = K a_1$ is sufficient to reduce the coning angle to zero.

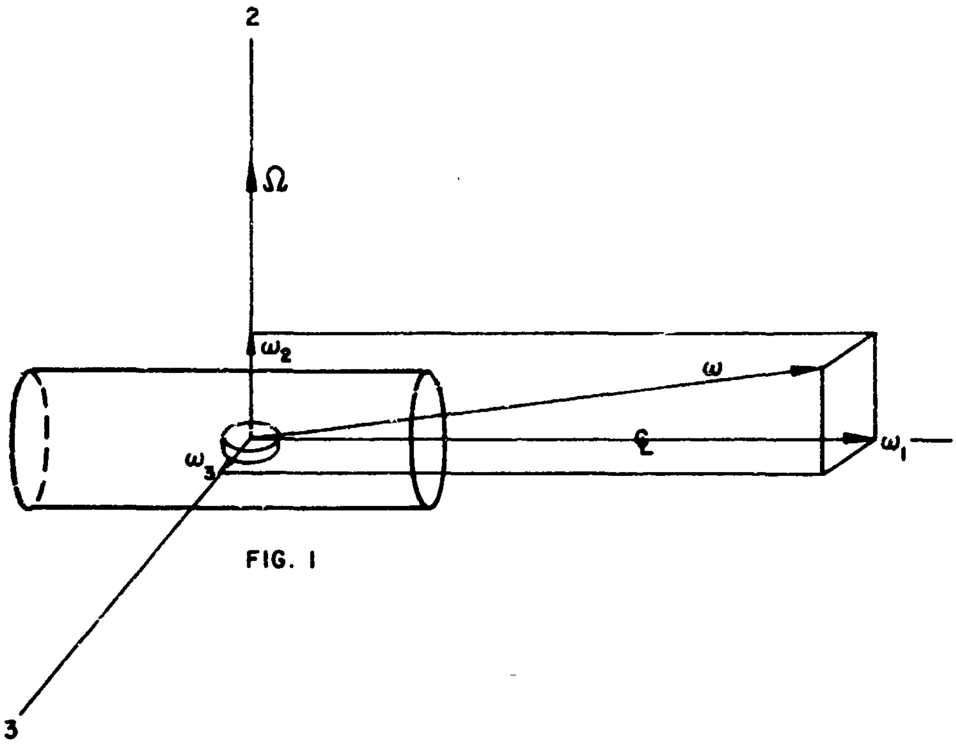

FIG. 1

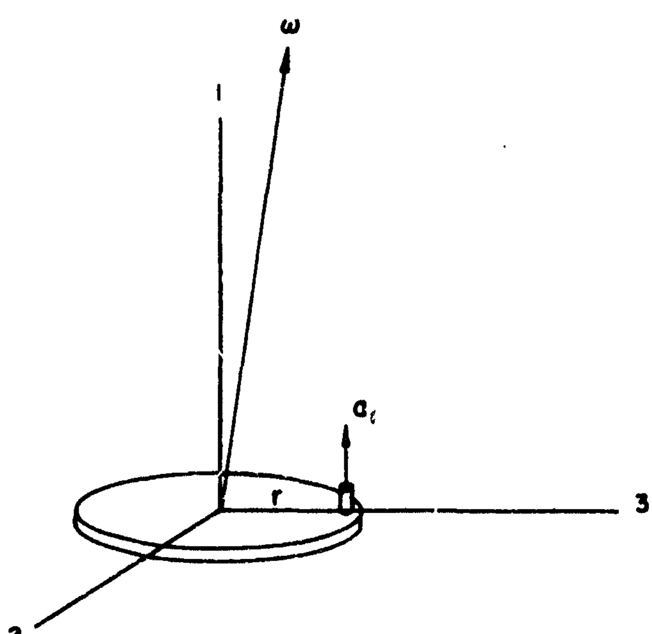

FIG. 2